BO MON TOAN ÖNG DUNG - ÑHBK

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# TOAN 1

# GIAI TÍCH HAM MOT BIEN

- BAI 7: KY NANG KHAI TRIEN TAYLOR


- TS. NGUYEN QUOC LAN (12/2007)

# KHAI TRIEN CÔ BAN: MU, LGIAC, HYPERBOLIC

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Tö khai trien ham y = $e^x$ $\Rightarrow$ Khai trien sinx, cosx, sinhx, coshx

$$e^x = 1 + x + \frac{x^2}{2!} + \ldots$$

Mu chan

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots + \frac{(-1)^n x^{2n}}{(2n)!} + o\left(x^{2n+1}\right),\ x \to 0$$

Mu le

$$\sin x = x - \frac{x^3}{3!} + \ldots + \frac{(-1)^n x^{2n+1}}{(2n+1)!} + o\left(x^{2n+2}\right),\ x \to 0$$

Tööng tö nhö $\sin x, \cos x$ nhöng khong ñan dau $\to \mathrm{sh}x, \mathrm{ch}x$

$$\mathrm{sh}x = x + \frac{x^3}{3!} + \ldots + \frac{x^{2n+1}}{(2n+1)!} + o\left(x^{2n+2}\right),\ \mathrm{ch}x = 1 + \frac{x^2}{2!} + \ldots + \frac{x^{2n}}{(2n)!} + o\left(x^{2n+1}\right)$$

$$\mathrm{tg}x = x + \frac{x^3}{3} + o\left(x^4\right),\ x \to 0$$

Chu y phan dö cosx, sinx, chx, shx: o nho cua so hang bò triet tieu!

# KHAI TRIEN CÔ BAN: LUY THÖA, 1/(1 ± x), LN(1 + x)

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Ham nghòch ñao – inverse function (Tong cap soi nhan):

$$\frac{1}{1-x}=1+x+\cdots+x^n+o\left(x^n\right),\ \frac{1}{1+x}=1-x+x^2+\cdots+(-1)^n x^n+o\left(x^n\right)$$

Tong quat: Ham luy thöa $(1 + x)^{\alpha}$ → Nhò thöc Newton $(1 + x)^n$

$$(1+x)^{\alpha}=1+\alpha x+\frac{\alpha(\alpha-1)}{2!}x^2+\cdots+\frac{\alpha\cdots(\alpha-n+1)}{n!}x^n+o\left(x^n\right)$$

VD: Khai trien MacLaurint ham $f(x)=\sqrt[3]{1+x}$ ñen cap 3

Giai: $(1+x)^{\frac{1}{3}}=1+\frac{x}{3}+\frac{1}{3}\left(\frac{1}{3}-1\right)\frac{x^2}{2!}+\frac{1}{3}\left(\frac{1}{3}-1\right)\left(\frac{1}{3}-2\right)\frac{x^3}{3!}+o\left(x^3\right),\ x\to 0$

ln(1 + x): $\int 1/(1+x)$

→ $x^n/n$, ñan dau

$$\ln(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}+\cdots+\frac{(-1)^{n-1}}{n}x^n+o\left(x^n\right)$$

# BANG KHAI TRIEN CAC HAM CÔ BAN: 7 HAM

| Ham | Khai trien | Phan dö Lagrange |
|---|---|---|
| $e^x$ | $1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots+\frac{x^n}{n!}$ | $\frac{e^c}{(n+1)!}x^{n+1}$ |
| $\cos x$ | $1-\frac{x^2}{2!}+\frac{x^4}{4!}-\ldots+(-1)^n\frac{x^{2n}}{(2n)!}x^{2n}$ | $\frac{\cos(\sin)c}{(2n+2)!}x^{2n+2}$ |
| $\sin x$ | $x-\frac{x^3}{3!}+\frac{x^5}{5!}-\ldots+(-1)^n\frac{x^{2n+1}}{(2n+1)!}x^{2n+1}$ | $\frac{\cos(\sin)c}{(2n+3)!}x^{2n+3}$ |
| $\frac{1}{1+x}$ | $1-x+x^2-x^3+\ldots+(-1)^n x^n$ | $(-1)^{n+1}\frac{1}{(1+c)^{n+2}}x^{n+1}$ |
| $\frac{1}{1-x}$ | $1+x+x^2+x^3+\ldots+x^n$ | |
| $(1+x)^\alpha$ | $1+\alpha x+\frac{\alpha(\alpha-1)}{2!}x^2+\ldots+\frac{\alpha\ldots(\alpha-n+1)}{n!}x^n$ | |
| $\ln(1+x)$ | $x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\ldots+(-1)^{n+1}\frac{x^n}{n}$ | |

# PPHAP KHTRIEN MACLAURINT: TONG, HIEU, TÍCH

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Ñöa ham can khai trien ve dang tong, hieu, tích (ñham, tphan) cac ham cô ban. Ap dung kh/tr MacLaurint cô ban

VD: Khai trien ML ñen cap 3: $f(x)=e^{x}+\dfrac{2}{1-x}-5\ln(1+x)$

Giai: $f(x)=\left(1+x+\dfrac{x^{2}}{2}+\ldots\right)+2\left(1+x+x^{2}+\ldots\right)-5\left(x-\dfrac{x^{2}}{2}+\ldots\right)+o\left(x^{3}\right)$

VD: Khai trien MacLaurint ñen cap 3: $f(x)=\cos x\cdot\cosh x$

Giai: $f(x)=\left(1-\dfrac{x^{2}}{2!}+o\left(x^{3}\right)\right)\left(1+\dfrac{x^{2}}{2!}+o\left(x^{3}\right)\right)=1+o\left(x^{3}\right),\ x\to 0$

Chu y: Co the söû dung ca ñao ham, tích phan (coi chöng C!)

VD: Khai trien ML ñen cap 2: $f(x)=\ln\left(x+\sqrt{1+x^{2}}\right)+1$

# KHTRIEN MACLAURINT HAM THÖÔNG: DUNG 1/(1 ± x)

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Vôi thöông (ty so, phan so) 2 ham so: Dung $\dfrac{1}{1 \pm x}$

Chu y: Ô mau so bat buoc phai xuat hien so 1!

VD: Khai trien MacLaurint $a/\ \dfrac{e^x}{2+x}$, cap 2 b/ $\dfrac{1}{\cos x}$, cap 3

Giai: $a/\ e^x \cdot \dfrac{1}{2} \cdot \dfrac{1}{1+x/2} = \dfrac{1}{2}\left(1+x+\dfrac{x^2}{2!}+o\left(x^2\right)\right)\left(1-\dfrac{x}{2}+\dfrac{x^2}{4}+o\left(x^2\right)\right)$

$$\text{b/ } \frac{1}{\cos x} = \frac{1}{1-\left(x^2/2!+o\left(x^3\right)\right)} = 1+\left(\frac{x^2}{2}+o\left(x^3\right)\right)+\left(\frac{x^2}{2}+o\left(x^3\right)\right)^2+\dots$$

VD: Khai trien MacLaurint ñen cap 2 $f(x) = \dfrac{1}{x^2-4x+3}$

Giai: $f(x) = \dfrac{1}{(x-1)(x-3)} = \dfrac{1}{2}\left[\dfrac{1}{x-1} - \dfrac{1}{x-3}\right] = \dfrac{1}{2}\left[\dfrac{1}{3} \cdot \dfrac{1}{1-x/3} - \dfrac{1}{1-x}\right]$

# KHAI TRIEÅN MACLAURINT VÔÙI HAØM HÔÏP

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Haøm hôïp f(u(x)): Khai trieån laàn löôït töøng böôùc. Ñaàu tieân khai trieån MacLaurint u(x), sau ñoù khai trieån f(u) & caét ñeán luyõ thöøa ñöôïc yeâu caàu (Coù theå ñoåi thöù töï).

Chuù yù quan troïng: Luoân kieåm tra ñieàu kieän u(0) = 0!

VD: Khai trieån MacLaurint $a/\ \sin\left(x^2\right)\quad b/\ \sqrt{\cos x}$ ñeán caáp 4

Giaûi: $a/\ u = x^2 \ \& \ u(0) = 0 \Rightarrow \sin u = u - \dfrac{u^3}{3!} + \ldots = x^2 + o\left(x^4\right)$

$$b/\ \sqrt{1 - \left(\frac{x^2}{2} - \frac{x^4}{24} + o\left(x^4\right)\right)} = \left[1 + \underbrace{\left(-\frac{x^2}{2} + \frac{x^4}{24} + o\left(x^4\right)\right)}_{u}\right]^{1/2} = 1 + \frac{1}{2}u + \ldots$$

VD (caûnh giaùc!): Khtrieån MacLaurint y = ln(2 + x) ñeán caáp 2

## KHAI TRIEN TAYLOR QUANH x – $x_0$: ÑÖA VE KTR ML

Khai trien Taylor f(x) quanh x = $x_0$: Ñoi bien t = x – $x_0$ va sö dung khai trien Mac Laurint cho ham f(t)

Cach 2: Bien ñoi ñe (x – $x_0$) xuat hien tröc tiep trong ham so!

VD: Khai trien Taylor ham $f(x)=\dfrac{1}{x}$ quanh $x_0=2$ ñen cap 3

Giai: Cach 1: t = x – 2 $\Rightarrow$ $$f(x)=\frac{1}{x}=\frac{1}{t+2}=\frac{1}{2}\cdot\frac{1}{1+t/2}=\frac{1}{2}\left(1+\frac{t}{2}+\dots\right)$$

Cach 2: Tao (x – 2) trong ham $$f(x)=\frac{1}{(x-2)+2}=\frac{1}{2}\cdot\frac{1}{1+(x-2)/2}$$

VD: Khai trien Taylor ham $f(x)=\sqrt[3]{x}$ quanh $x_0=-8$ ñen cap 2

Giai: $$\sqrt[3]{(x+2)-8}=-2\left[1-\frac{(x+2)}{8}\right]^{1/3}=-2\left(1+\frac{1}{3}\cdot\left(\frac{-(x+2)}{8}\right)+\dots\right)$$

## ÖNG DUNG KT TAYLOR. TÌM GIÔI HAN

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**Tìm lim: Khai trien ML vôi phan dö Peano + Ngat boi VCB**

VD: Tính $\lim\limits_{x\to0}\dfrac{x-\sin x}{x^3}=\lim\limits_{x\to0}\dfrac{x-\left[x-\dfrac{x^3}{6}+o\left(x^4\right)\right]}{x^3}=\lim\limits_{x\to0}\dfrac{\dfrac{x^3}{6}+o\left(x^4\right)}{x^3}$

VD: Tìm $\lim\limits_{x\to0}\dfrac{\sin 3x+4\sin^3 x-3\ln(1+x)}{\left(e^x-1\right)\sin x}=\lim\limits_{x\to0}\dfrac{\sin 3x-3\ln(1+x)}{x^2}$

VD: Tìm $\lim\limits_{x\to0}\left[\dfrac{1}{x(1+x)}-\dfrac{\ln(1+x)}{x^2}\right]=\lim\limits_{x\to0}\dfrac{x-(1+x)\ln(1+x)}{x^2(1+x)}$

$$=\lim\limits_{x\to0}\left[\frac{x-\ln(1+x)}{x^2(1+x)}-\frac{x\ln(1+x)}{x^2(1+x)}\right]$$

(SGK/80) $\lim\limits_{x\to\infty}\left[x-x^2\ln\left(1+\dfrac{1}{x}\right)\right]$

# ÖNG DUNG KT TAYLOR. TÍNH GAN ÑUNG

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Tính gan ñung & öôc löông sai so: phan dö Lagrange

$$f(x) \approx \sum_{k=0}^{n} \frac{f^{(k)}(x_0)}{k!}(x-x_0)^k \,,\ \Delta = |R_n| = \frac{\left|f^{(n+1)}(c)\right|}{(n+1)!}|x-x_0|^{n+1} \,,\ c \in (x_0, x)$$

VD: Tính gan ñung gia trò so e vôi ñoi chính xac $10^{-4}$ (SGK/79)

Giai: $$e = \underbrace{1 + \frac{1}{1!} + \frac{1}{2!} + \ldots + \frac{1}{n!}}_{S} + \frac{e^c}{(n+1)!} \,,\ c \in (0,1) \Rightarrow e \approx S,\ \Delta \le \frac{3}{(n+1)!}$$

Töông tö: Can chon bao nhieu so hang trong khai trien ham y $= e^x$ ñe co the xap xæ e vôi ñoi chính xac $10^{-4}$

VD: Goc x nao cho phep xap xæ $\sin x \approx x$ vôi ñoi chính xac $10^{-4}$

# VI PHAN

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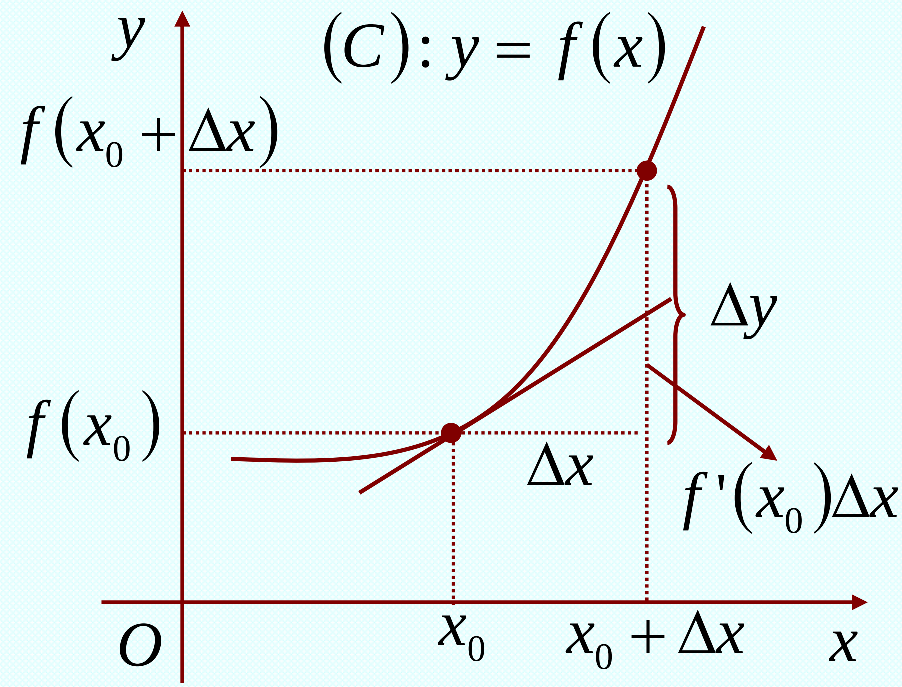
Ham khai vi tai $x_0$ $\Leftrightarrow$ $\Delta y = A\Delta x + o(\Delta x)$, $\Delta x \to 0$ : Soi gia ham soi bieu dien tuyen tính theo $\Delta x$ va voi cung be bac cao cua $\Delta x$

> Vi phan: $dy = A\Delta x = f'(x)dx$

Nhan xet: Ham coi ñaio ham $\Leftrightarrow$ Coi vi phan: Ham khai vi




1/ C: hang soi $\Rightarrow$ $dC = 0$

& $d(Cy) = Cdy$

2/ Vi phan tong, hieu, tích, thöông:

$$d(u \pm v) = du \pm dv \qquad d\left(\frac{u}{v}\right) = \frac{vdu - udv}{v^2}$$
$$d(uv) = vdu + udv$$

# VI PHAN HAM HÔP

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Vi phan cap 1: $\left.\begin{bmatrix} y = f(x), x: \text{bien ñoc lap} \\ y = f(x), x = x(t): \text{ham hôp} \end{bmatrix}\right\} \Rightarrow dy = y'dx$

$\Rightarrow$ Vi phan cap 1: bat bien!

VD: Tính dy cua a/ y = sinx b/ y = sinx, x = cost

Giai: $b/dy = \cos x dx = -\cos x \sin t dt$ hoac $y = \sin(\cos t) \Rightarrow dy = \ldots$

Vi phan cap cao: $x: \text{Bien ñoc lap} \Rightarrow d^2y = f''dx^2, d^3y = \ldots$

$$y = f(x), x = x(t) \Rightarrow d^2y = f''dx^2 + f'd^2x \quad \left(d^2x = x''dt^2\right)$$

VD: Tính $d^2y$: a/ y = arctgx b/ y = arctgx, x = sint

ÑS: $a/d^2y = -\dfrac{2x}{\left(1+x^2\right)^2}dx^2 \quad b/d^2y = y''dx^2 - \dfrac{\sin t}{1+x^2}dt^2$