Trao đổi trực tuyến tại:

http://www.mientayvn.com/chat_box_toan.html

BO MON TOAN ÖNG DUNG - ÑHBK

TOAN 1

GIAI TÍCH HAM MOT BIEN

- BAI 5: ÑAO HAM

- TS. NGUYEN QUOC LAN (11/2007)

NOÄI DUNG

1- ÑÒNH NGHÓA ÑAÏO HAØM

2- DUNG ÑÒNH NGHÓA TÍNH ÑAÏO HAØM: HAØM KHOÂNG SÔ CAÁP (HAØM GHEÙP) – ÑAÏO HAØM 1 PHÍA

3- ÑAÏO HAØM HAØM AÅN

4- ÑAÏO HAØM LÖÔÏNG GIAÙC NGÖÔÏC

5- ÑAÏO HAØM HAØM THEO THAM SOÁ

6 – ÑAÏO HAØM CAÁP CAO

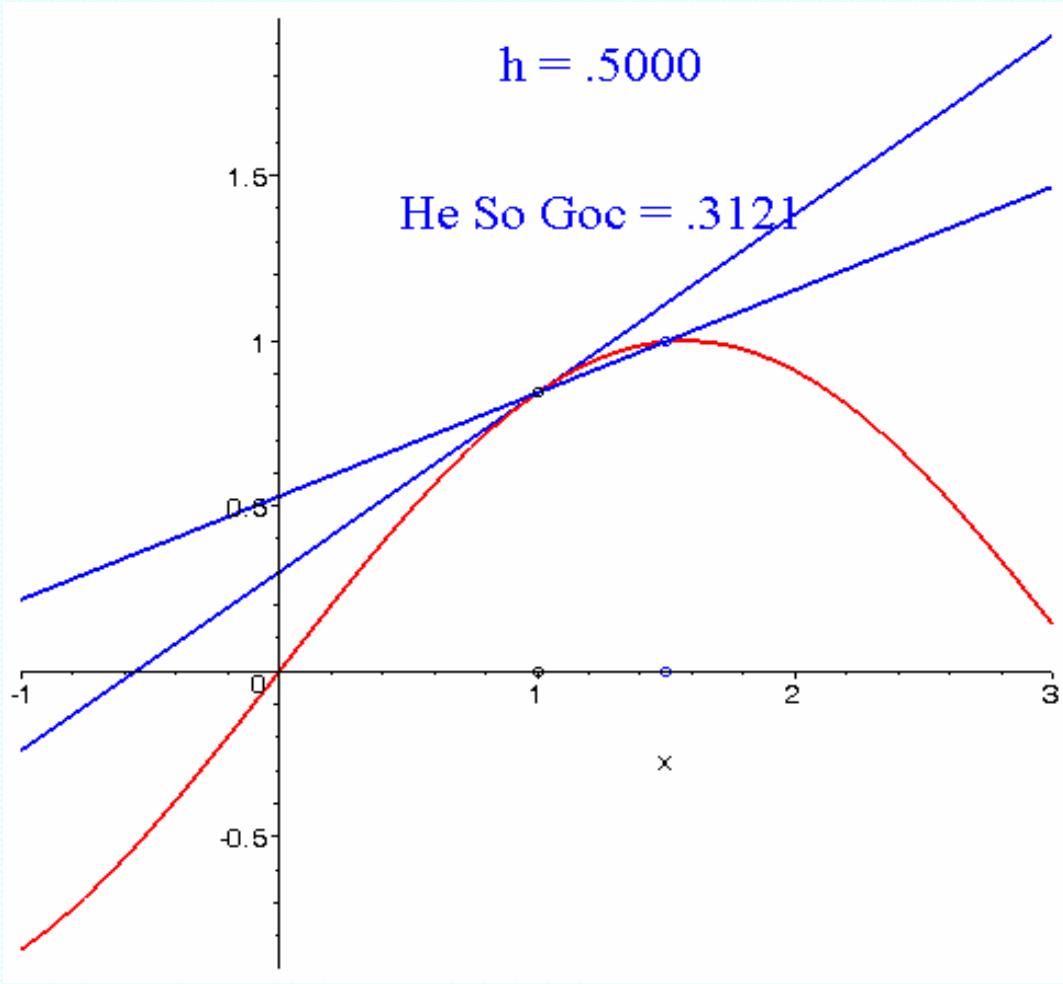
ÑAÏO HAÌM

$$f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} = \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

Yù nghóa hình hoïc: Heä soá goùc tieáp tuyeán cuûa ñoà thò (C) y = f(x) taïi tieáp ñieåm $M(x_0, f(x_0))$

Haøm coù ñaïo haøm taïi $x_0 \Rightarrow$ Lieân tuïc taïi x_0.

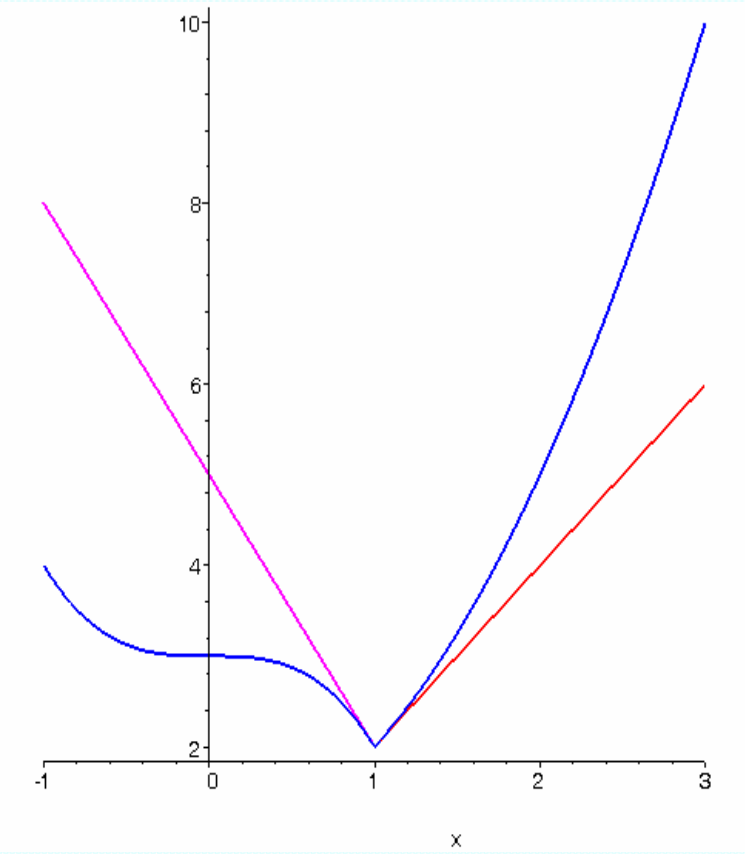
Ngöôïc laïi: SAI!



HAM GHEP, TRÒ TUYET: ÑAO HAM MOT PHÍA

Ñao ham phai: $f'(x_0+) = \lim_{\Delta x \to 0+} \dfrac{f(x_0+\Delta x) - f(x_0)}{\Delta x}$ (i.e $\Delta x > 0$)

Ñao ham trai: $f'(x_0-) = \lim_{\Delta x \to 0-} \dfrac{f(x_0+\Delta x) - f(x_0)}{\Delta x}$ (i.e $\Delta x < 0$)



Ham y = f(x) coù ñao ham höu han tai $x_0 \Leftrightarrow f'(x_0+) = f'(x_0-)$

VD: Tính ñao ham tai $x_0 = 1$

$$f(x) = \begin{cases} x^2, & x \le 1 \\ 2x - 1, & x > 1 \end{cases}$$

VD: $f(x) = |x|$, $x_0 = 0$

KHI NAO DUNG ÑAO HAM 1 PHÍA?

Ñao ham ham sô cap (xac ñònh qua 1 bieu thöc): bang ñao ham cô ban + ñao ham tong, hieu, tích, thöông, hôp

Ñao ham ham khong sô cap (≥ 2 bieu thöc): ñònh nghóa & dung ñao ham trai, ñao ham phai

VD: Tìm a, b ñe ham so sau co ñao ham tai $x_0 = 0$

$$f(x)=\begin{cases} ax^2+bx+1, & x\geq 0 \\ a\sin x+b\cos x, & x<0 \end{cases}$$

Chu y: Nen kiem tra tröôc ñieu kien lien tuc

VD: Tính ñao ham tai $x_0 = 0$ cua ham

$$f(x)=\begin{cases} x^2\sin\dfrac{1}{x}, & x\neq 0 \\ 0, & x=0 \end{cases}$$

TÍNH ÑAO HAM HAM SÔ CAP

Bang ñao ham cac ham sô cap cô ban: töi xem lai

Ñao ham	Ñao ham ham hôp
$(C)' = 0$	
$(x^{\alpha})' = \alpha x^{\alpha-1}$	$(u^{\alpha})' = \alpha u^{\alpha-1}.u'$
$(1/x)' = -1/x^2$	$(1/u)' =$
$\left(\sqrt{x}\right)' = 1/2\sqrt{x}$	$\left(\sqrt{u}\right)'$
$(\sin x)' = \cos x$	$(\sin u)' =$
$(\cos x)' = -\sin x$	$(\cos u)' =$
$(\tg x)' = 1/\cos^2 x = 1 + \tg^2 x$	$(\tg u)' =$
$(\cotg x)' = -1/\sin^2 x =$	$(\cotg u)' =$
$(e^x)' = e^x,\ (a^x)' = a^x \ln a$	$(e^u)' =$
$(\ln x)' = 1/x,\ (\log_a x) = 1/(x \ln a)$	$(\ln u)' =$

QUY TAÉC TÍNH ÑAÏO HAØM

Quy taéc ñaïo haøm toång, hieäu, tích, thöông: töï xem laïi

$$(u \pm v)' = u' \pm v' \qquad (Cu)' = Cu' \qquad (uv)' = u'v + v'u$$

$$(uvw)' = u'vw + uv'w + uvw' \qquad \left(\frac{u}{v}\right)' = \frac{u'v - v'u}{v^2}$$

Ñaïo haøm haøm hôïp: Quy taéc daây xích!

$y = f(u), u = u(x) : y = f(u(x)) \Rightarrow y'_x = y'_u \cdot u'_x$: Xuaát hieän u'!

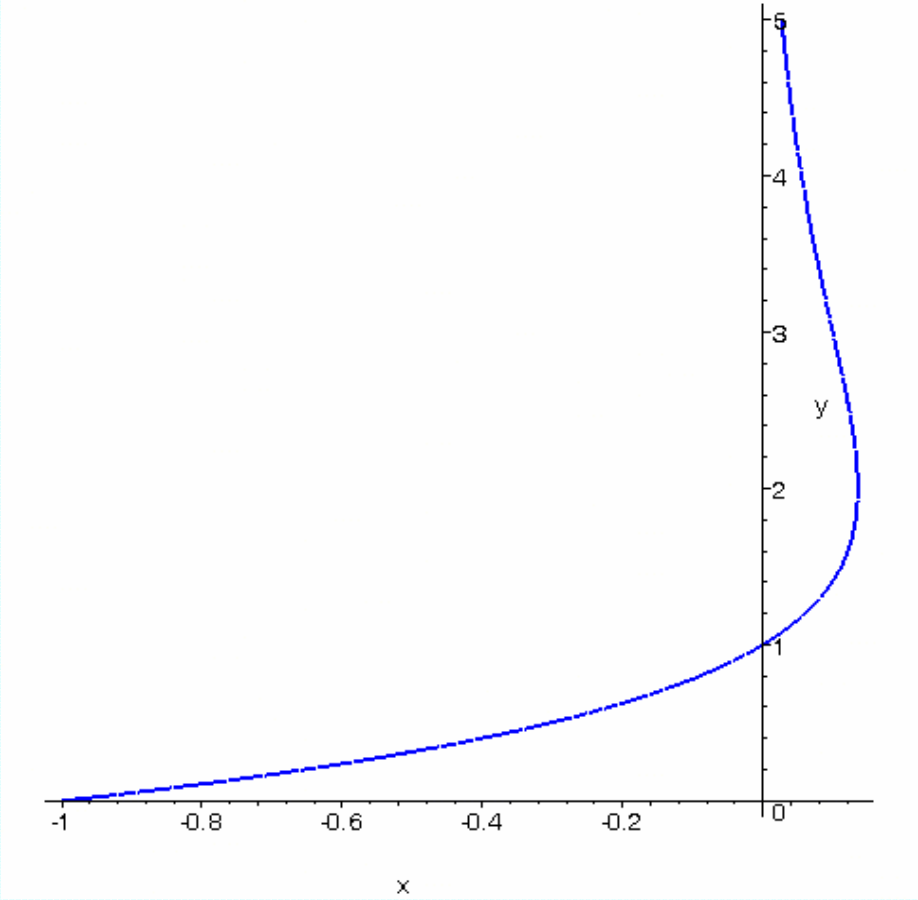
VD: Cho y = f(x^2). Tính caùc ñaïo haøm y', y''

$y = f(x)^{g(x)} \Rightarrow$ log (cô soá e) hoaù 2 veá. VD: $y = \left(1 + \frac{1}{x}\right)^{x^2} \Rightarrow y' = ?$

ÑAO HAM HAM AN

Ham an : $F(x,y) = 0 \ \forall\ x \in [a, b] \Rightarrow y = y(x) \ \forall\ x \in [a, b]$

VD : Ham an y = y(x) xac ñònh töø phöông trình $y = 1 + xe^y$



Tính y': Ñao ham tröc tiep 2 veá theo x, chuù y y = y(x) roài giai phöông trình an y'

VD ñang xeùt : $y'_x = \dfrac{e^y}{1 - xe^y}$

VD : Ñao ham y'(0) cua ham an

$$x^3 + \ln y - x^2 e^y = 0 \Rightarrow y'(x) =$$

$$y(0) = \qquad \Rightarrow \quad y'(0) =$$

ÑAO HAM HAM LÖÔNG GIAC NGÖÔC – HYPERBOLIC

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y = f(x) → ham ngöôc x = g(y). Taïi $y_0 = f(x_0)$:

$$g'(y_0) = \frac{1}{f'(x_0)} \Rightarrow \left(f^{-1}(y)\right)' = \frac{1}{f'(x)}$$

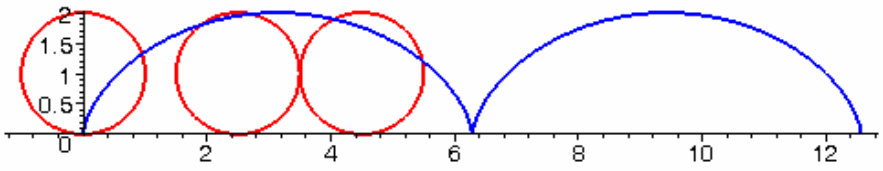
Gnhôù: $(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$; $(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$; $(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

(arcsinx)' = $1/\sqrt{1-x^2}$	(arcsinu)' = $u'/\sqrt{1-u^2}$
(arccosx)' = $-1/\sqrt{1-x^2}$	(arccosu)' = $-u'/\sqrt{1-u^2}$
(arctgx)' = $1/(1+x^2)$	(arctgu)' = $u'/(1+u^2)$
(arccotgx)' = $-1/(1+x^2)$	(arccotgu)' = $-u'/(1+u^2)$
(shx)' = chx	(shu)' = u' . chu
(chx)' = shx	(chu)' = u' . shu
(thx)' = $1/ch^2x = 1 - th^2x$	(thu)' = $u'/\cosh^2 u$
(cothx)' = $-1/sh^2x = 1 - coth^2x$	(cothu)' = $-u'/\sinh^2 u$

ÑAO HAM HAM THEO THAM SO

Ham theo tham so : x = x(t), y = y(t) $\Rightarrow$ y = y(x)

VD : Ham biеu dien ñöông cycloid x = a(t – sint), y = a(1 – cost)



P/phap: Ñöa ve ñ/ham theo t!

$$y'_x = \frac{y'(t)}{x'(t)};\ y''_x = (y'_x)'_x = \frac{(y'_x)_t}{x'_t}$$

Ñöông cycloid $y'_x = \dfrac{\sin t}{1-\cos t}$

VD : Tham so hoa ñöông elip & viet p/trình tiep tuyen:

$$\begin{cases} x = a\sin t \\ y = b\cos t \end{cases} \Rightarrow y'_x = \frac{y'_t}{x'_t} = \frac{(b\cos t)'}{(a\sin t)'}$$

ÑAO HAM CAP CAO

Ñham cap 2: y''(x) = [y'(x)]' . ÑH cap n: $y^{(n)}(x) = [y^{(n-1)}(x)]'$

Ky hieu: $\frac{d^n y}{dx^n}$

Mot so ñao ham cap cao cô ban:

$$\left(e^x\right)^{(n)} = e^x \qquad \left(a^x\right)^{(n)} = a^x \ln^n a$$

$$(\sin x)^{(n)} = \sin\left(x + n\frac{\pi}{2}\right) \qquad (\sin(ax+b))^{(n)} = a^n \sin\left(ax + b + n\frac{\pi}{2}\right)$$

$$\left[(ax+b)^{\alpha}\right]^{(n)} = a^n \alpha(\alpha-1)\ldots(\alpha-n+1)(ax+b)^{\alpha-n}$$

$$(\ln(ax+b))^{(n)} = \frac{(-1)^{n-1} a^n (n-1)!}{(ax+b)^n}$$

KYÕ NAÊNG TÍNH ÑAÏO HAØM CAÁP CAO

Phaân tích haøm veà daïng "toång" caùc haøm ñôn giaûn

VD: $f(x) = \dfrac{1}{x^2 - 1}$

VD: $f(x) = \sin^2 x$

Lebnitz : $(uv)^{(n)} = \sum_{k=0}^{n} C_n^k u^{(k)} v^{(n-k)} = C_n^0 uv^{(n)} + C_n^1 u'v^{(n-1)} + \ldots + C_n^n u^{(n)} v$

VD: $f(x) = x^2 e^x$

Toång quaùt: f(x) = u.v, u – ña thöùc baäc m $\Rightarrow$ Caùc ñaïo haøm $u^{(k)} = 0 \ \forall \ k > m \Rightarrow$ Toång $u^{(k)}v^{(n-k)}$ chæ goàm vaøi thöøa soá: tính ñôn giaûn!