BO MON TOAN ÖNG DUNG - ÑHBK

PHÖÔNG PHAP TÍNH – HK2 0506

CHÖÔNG 1

GIAI GAN ÑUNG PHÖÔNG TRÌNH PHI TUYEN $f(x) = 0$

- TS. NGUYEN QUOC LAN (02/2006)

NOI DUNG

1– KHAI NIEM TONG QUAT. CONG THÖC SAI SO

2– PHÖÔNG PHAP CHIA ÑOI

3– PHÖÔNG PHAP LAP ÑÔN

4– PHÖÔNG PHAP NEWTON (TIEP TUYEN)

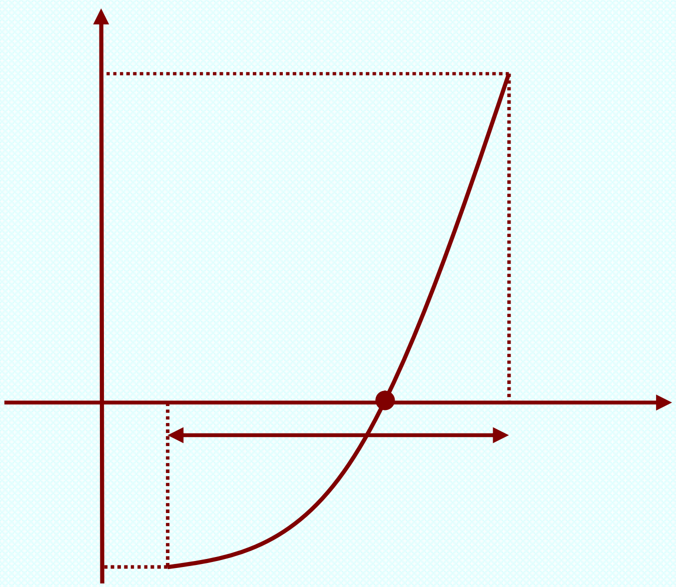
5– HE PHÖÔNG TRÌNH PHI TUYEN. PHÖÔNG PHAP NEWTON – RAPHSON.

1. KHAI NIEM TONG QUAT – CONG THÖC SAI SO

Phöông trình f(x) = 0 (1), f: ham so lien tuc, co ñao ham

Khoang cach ly nghiem: Ñoan [a, b] (hoac khoang (a, b)), tren ño phöông trình (1) co nghiem α duy nhat

VD: Phöông trình x – cosx = 0 co khoang cach ly nghiem:



ÑK ñu: [a, b] la KCLN cua (1) khi

- Ñao ham f' khong ñoi dau tren ñoan (hoac khoang) (a,b)
- f(a).f(b) < 0 (gia trò 2 ñau trai dau)

Tìm KCLN: Tính f', lap bang bien thien; Cach 2: Ño thò (may!)

CONG THÖC SAI SO

<u>Cong thöc sai so tong quat</u>: Phöông trình f(x) = 0 (1) vôi nghiem chính xac α tren khoang cach ly nghiem [a, b]

$$\begin{cases} \bar{x} \in [a,b] : \text{Nghiem gan ñung ña biet} \\ |f'(x)| \ge m_1 > 0 \ \forall \ x \in [a,b] \Leftrightarrow m_1 = \min\limits_{[a,b]} |f'(x)| \end{cases} \Rightarrow |\bar{x} - \alpha| \le \frac{|f(\bar{x})|}{m_1}$$

VD: P/trình f(x) = x – cosx = 0 co khoang cach ly nghiem [0,1]

Neu chon nghiem gan ñung $\bar{x} = 0.739 \Rightarrow \Delta_{\bar{x}} : \begin{cases} a/0.00014\,? \\ b/0.00015\,? \end{cases}$

Giai:

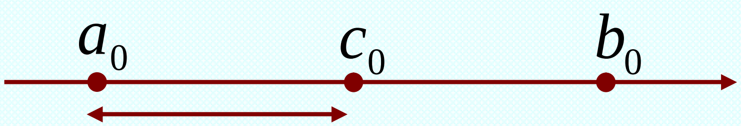
Ghi nhô: Sai so luon lam tron len

PHÖÔNG PHAP CHIA ÑOI

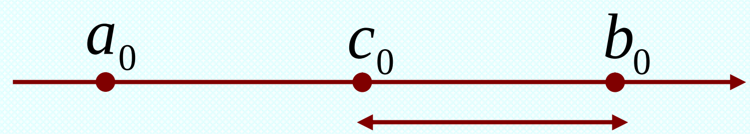
Y töông: Lien tuc chia ñoi khoang cach ly nghiem

f(x) = 0 tren KCL nghiem [a, b]. Ky hieu: $a_0 = a$, $b_0 = b \Rightarrow$

$f(a_0).f(b_0) < 0$. Chia ñoi: $c_0 = (a_0 + b_0)/2 \Rightarrow$ KCL nghiem môi?



$f(a_0).f(c_0) < 0$: KCL môi $[a_0, c_0]$



$f(c_0).f(b_0) < 0 \rightarrow [c_0, b_0]$

Döng vôi nghiem xap xæ $\bar{x} = c_n$ (trung ñiem ô hang thö n)

Cong thöc sai so: $|\bar{x} - \alpha| \le \dfrac{b-a}{2^{n+1}} \le \varepsilon \Leftrightarrow n \ge \dfrac{\log\left[\dfrac{(b-a)}{\varepsilon}\right]}{\log 2} - 1$

VÍ DU PHÖÔNG PHAP CHIA ÑOI

Xap xæ nghiem cua phöông trình f(x) = x – cosx = 0 tren khoang cach ly nghiem [0, 1] vôi sai so 0.2

Giai: Lap bang chöa moi ket qua trung gian can thiet

n	a_n	b_n	c_n	ε_n

Tìm n ñe co the xap xæ nghiem f(x) = x – cosx = 0 tren khoang cach ly nghiem [0, 1] bang phöông phap chia ñoi, sai so 10^{-8}

DAY LAP ÑÔN

Day lap ñôn: Day $\{x_n\}$ xac ñònh $x_{n+1} = \varphi(x_n)$, $\varphi(x)$: ham lap

VD: Kiem tra nhöng day sau coi la lap ñôn? Neu coi, viet ra ham lap φ. Tính 5 soi hang ñau cua day (x_0 bat ky). Tö ñoi, ñoan tính hoi tu? Tìm lien hei giöa giôi han day va ham lap φ

$$a/\ x_{n+1} = \cos\left(\frac{x_n}{10}\right)$$

$$b/\ y_{n+1} = \sqrt[3]{n y_n}$$

$$c/\ z_{n+1} = \frac{z_n}{15} + 1$$

n	x_n
0	
1	
2	

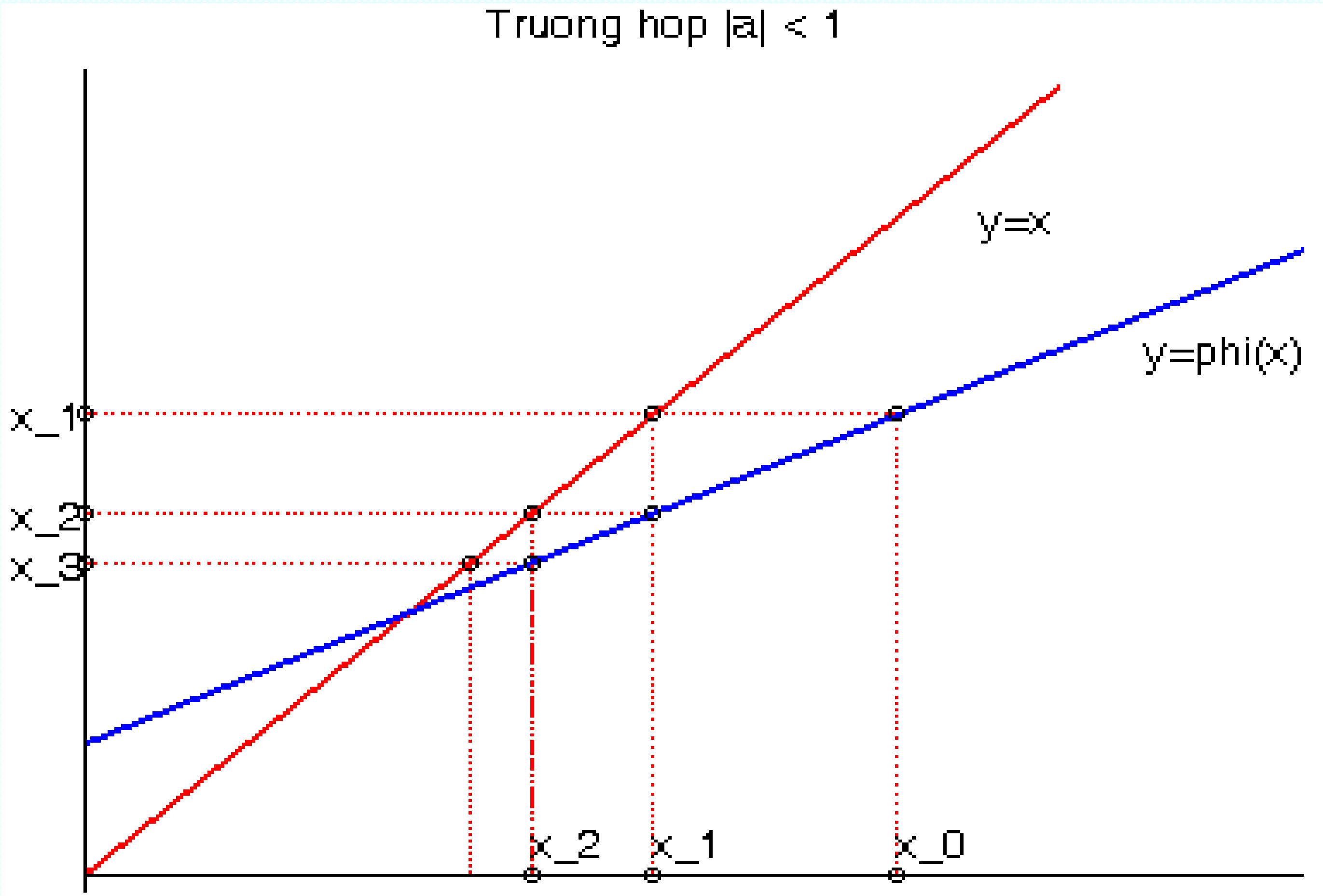
n	z_n
0	
1	
2	

Day lap ñôn $x_n = \varphi(x_{n-1})$ hoi tu ve $\alpha \Rightarrow \alpha$ la nghiem p/t $x = \varphi(x)$

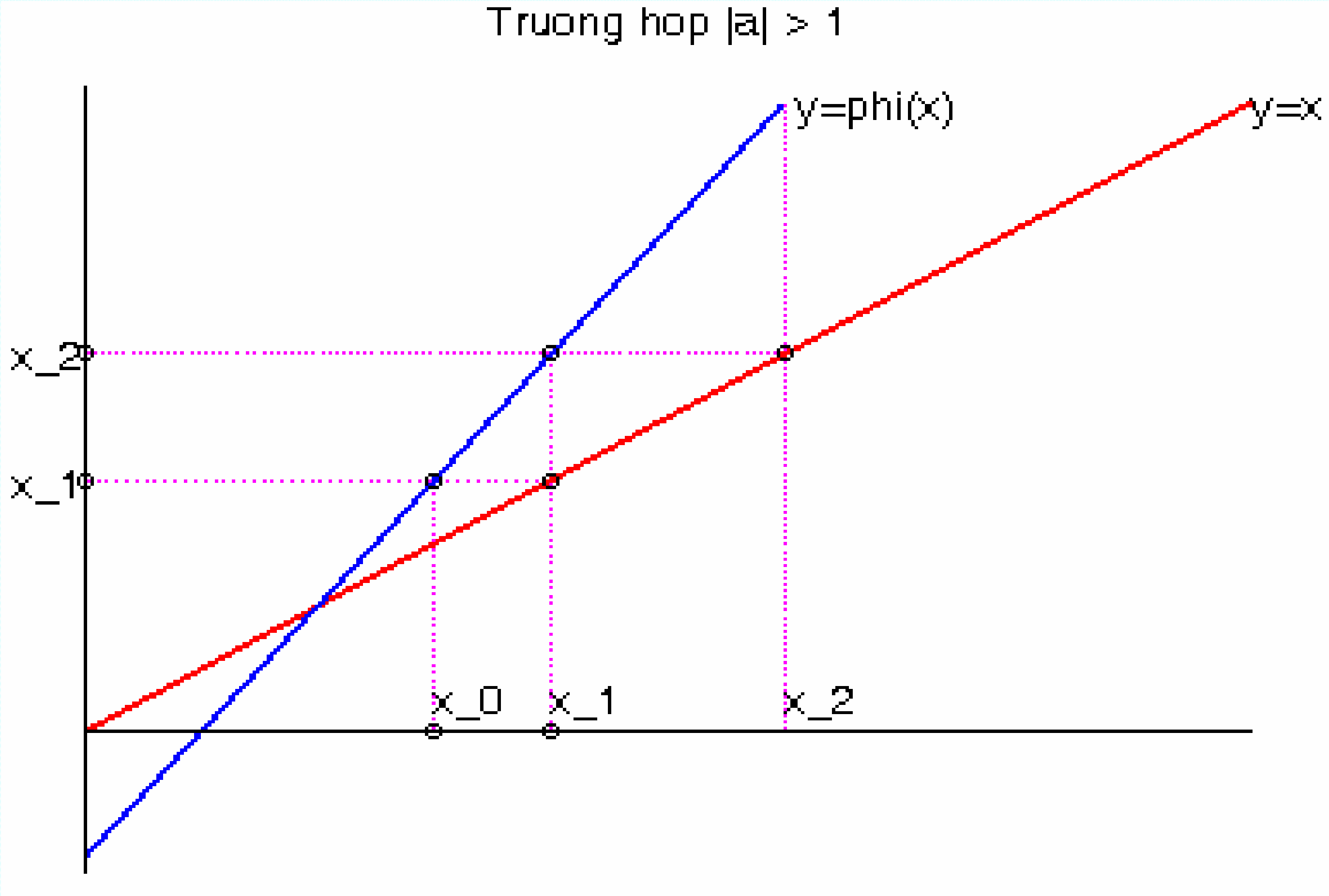
DAY LAP ÑÔN HOI TU

Minh hoa söi hoi tui cua day lap ñôn: $x_{n+1} = \varphi(x_n) = ax_n + b$

Day lap hoi tu ve nghiem p/trình: $x = \varphi(x) \Rightarrow \alpha = b/(1 - a)$



DAÕY LAÄP ÑÔN PHAÂN KYØ



Phaân kyø

Hoäi tuï khi daõy $\{x_n\}$ "co" laïi $\Rightarrow |x_2 - x_1| = |\varphi(x_1) - \varphi(x_0)| \le q|x_1 - x_0|$

HAØM CO

Haøm y = φ(x) $\boxed{\text{co}}$ treân [a, b] vôùi $\boxed{\text{heä soá co q} \Leftrightarrow}$ ∃ q, 0 < q < 1:

$$\forall\, x, y \in [a,b] : \; |\varphi(x) - \varphi(y)| \;\le\; q|x - y|$$

|φ'(x)| ≤ q < 1 ∀ x∈[a, b] ⇒ φ(x) co treân [a, b] vôùi heä soá co q

VD: Haøm y = x^2 co treân [-1/4, 1/4]???

VD: Trong nhöõng haøm sau ñaây, haøm naøo thoaû ñieàu kieän co?

Xaùc ñònh haèng soá q vôùi caùc haøm co ñoù

$$a/\ \varphi(x) = \cos x\ ,\ x \in [0,1]$$

$$b/\ \varphi(x) = \arcsin x\ ,\ x \in R$$

$$c/\ \varphi(x) = \frac{1}{x^2} + 1\ ,\ x \in [a,b],\ 0 < a < b$$

PHÖÔNG PHAP LAP ÑÔN

- Ph. trình f(x) = 0. Xac ñònh khoang cach ly nghiem [a, b]
- Ñöa pt f(x) = 0 ve dang lap ñôn $x = \varphi(x)$, φ co tren [a, b]

Lay x_0 bat ky $\in [a,b] \Rightarrow$ Day lap $x_{n+1} = \varphi(x_n) \to \alpha$

Chu y: Nhieu cach chon ham $\varphi \Rightarrow$ cang ñôn gian cang tot

Öôc löông sai so (q: he so co cua ham lap ñôn $\varphi(x)$)

Tien nghiem: $$|x_n - \alpha| \le \frac{q^n}{1-q}|x_1 - x_0|$$

Hau nghiem: $$|x_n - \alpha| \le \frac{q}{1-q}|x_n - x_{n-1}|$$

So lan lap toi thieu:

$$|x_n - \alpha| \le \varepsilon \Leftrightarrow$$

$$n \ge \frac{\log\left[\varepsilon(1-q)/|x_1 - x_0|\right]}{\log q}$$

VÍ DU PHÖÔNG PHAP LAP ÑÔN

Xap xæ nghiem ptrình f(x) = $x^3 + x - 1000 = 0$ vôi sai so 10^{-8}

Giai: Khoang cach ly nghiem

Lap ñôn: x = 1000 – x^3 = φ(x): Kiem tra ñieu kien co?

Xay döng ham lap môi: $x = \sqrt[3]{1000 - x} = \varphi(x)$ $\Rightarrow$ Ham co?

Day lap: $\begin{cases} x_0 \in [9,10] \\ x_{n+1} = \varphi(x_n) = \sqrt[3]{1000 - x_n} \end{cases}$

Sai so:

$$|x_n - \alpha| \le \frac{q}{1-q}|x_n - x_{n-1}|$$

n	x_n	ε_n
0		

CAÛI TIEÁN PHÖÔNG PHAÙP LAËP ÑÔN

Nhaän xeùt: $q = 0.0034 << 1 \Rightarrow$ Hoäi tuï raát nhanh

Vaán ñeà: Xaây döïng haøm φ vôùi $q << 1$?

Tìm soá laàn laëp ñeå xaáp xæ nghieäm $x - \cos x = 0$ treân [0,1] vôùi phöông phaùp laëp ñôn, $x_0 = 0$ vôùi sai soá 10^{-8}

Giaûi: Daïng laëp $x = \cos x = \varphi(x) \Rightarrow q =$

$x_0 = 0 \Rightarrow x_1 = \varphi(x_0) = 1$

Öôùc löôïng sai soá tieân nghieäm:

Caûi tieán toác ñoä: Laëp Newton $\quad f(x) = 0 \Leftrightarrow x = x - \dfrac{f(x)}{f'(x)} = \varphi(x)$

PHÖÔNG PHAP LAP NEWTON (TIEP TUYEN)

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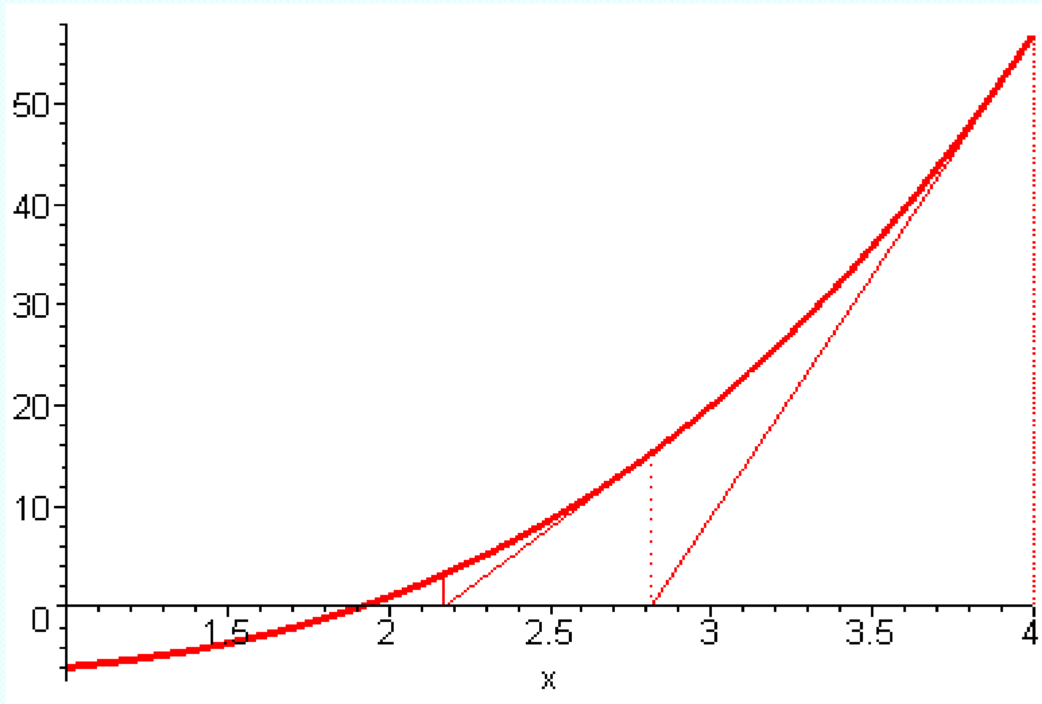
f(x) = 0 ⇔ Daïng laëp ñôn $x = g(x) = x - \dfrac{f(x)}{f'(x)}$: hoäi tuï nhanh

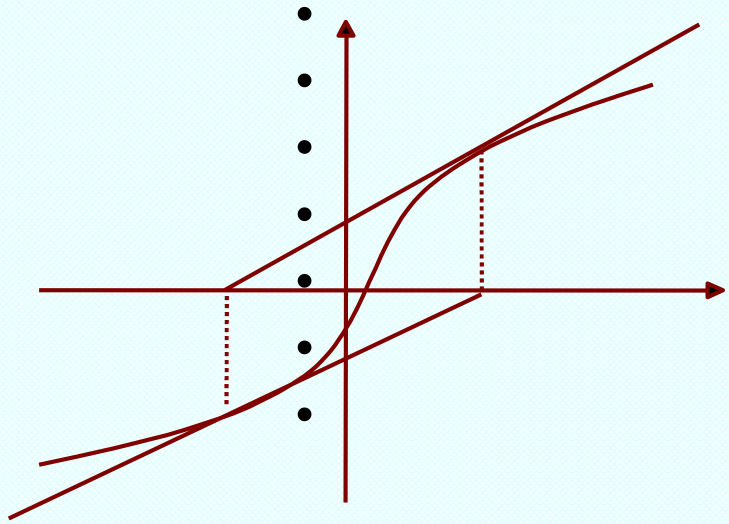
•Coâng thöùc laëp Newton: $x_{n+1} = g(x_n) = x_n - \dfrac{f(x_n)}{f'(x_n)}$

•Minh hoaï hình hoïc:



ÑIEU KIEN LAP NEWTON – SAI SO

Lap Newton that bai:



Ñieu kien hoi tu: 1/ Ñham f’, f” khong ñoi dau tren [a, b]

2/ Gia trò lap ban ñau thoa: $f(x_0) \cdot f''(x_0) > 0$ (ÑK Fourier)

•Öôc löông sai so : Cong thöc tong quat (chu yeu) hoac

$$|x_n - \alpha| \le \frac{M_2}{2m_1}(x_n - x_{n-1})^2,\ M_2 = \max_{[a,b]} |f''|$$

•(Phöc tap hôn)

VÍ DU LAP NEWTON – TIEP TUYEN

Giai xap xæ $f(x) = x - \cos x = 0$ tren $[0, 1]$, sai so 10^{-8}

1/ Kiem tra ñieu kien hoi tu

2/ Xay döng day lap:

Sai so :

n	x_n	ε_n
0	1	

HE PHI TUYEN – PP NEWTON – RAPHSON

Minh hoa : He 2 phöông trình, 2 an

$$\begin{cases} f_1(x_1, x_2) = 0 \\ f_2(x_1, x_2) = 0 \end{cases} \Leftrightarrow f(x) = 0 \, , \; f = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} : R^2 \to R^2 \, , \; x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Ky hieu ma tran f'(x) (ma tran Jacobi): $f'(x) = \begin{bmatrix} \dfrac{\partial f_1}{\partial x_1} & \dfrac{\partial f_1}{\partial x_2} \\ \dfrac{\partial f_2}{\partial x_1} & \dfrac{\partial f_2}{\partial x_2} \end{bmatrix}$ $\Rightarrow$ Co the tính "gia trò" f'($x^{(0)}$) tai "ñiem" $x^{(0)}$ cho tröôc

Ky hieu: $x^{(k)} = \left\lfloor x_1^k , x_2^k \right\rfloor$: Bo nghiem gan ñung ô böôc thö k

Xem $x^{(k)}$ ñaõ biet. Tính $x^{(k+1)}$: giai thuat Newton - Raphson

1/ Tính ma tran A = f'($x^{(k)}$) (thay $x^{(k)}$ vao) & vectô b = –f($x^{(k)}$)

2/ Giai he p/tr (bang may bo tui) Ah = b. Tính $x^{(k+1)} = x^{(k)} + h$

VÍ DU LAP NEWTON – RAPHSON VÔI HE PHI TUYEN

Tìm nghiem gan ñung $x^{(1)}$ cua he phi tuyen sau vôi 3 chö so le:

$$\begin{cases} f_1(x_1, x_2) \equiv x_1 + 3\ln x_1 - x_2^2 = 0 \\ f_2(x_1, x_2) \equiv 2x_1^2 - x_1 x_2 - 5x_1 + 1 = 0 \end{cases} \quad , x^{(0)} = [1.5, -1.5]^T$$

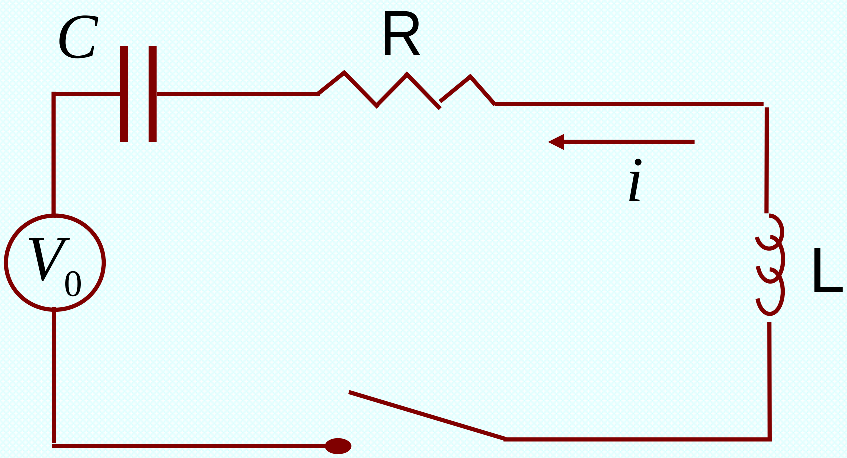
Giai: Ma tran A = f'(x)

b "nho": $x^{(k)}$ gan nghiem

n	$x^{(n)}$	Ma tran Jacobian A		Vectô $-f(x^{(n)})$	Vectô h
0	1.5				
	−1.5				

ÖNG DUNG THÖC TE: LY THUYET MACH

Mach ñien: Nguon (pin) V_0, Ñien trô R, Tu C, Cam öng L



$$U_R = iR,\ U_L = L\frac{di}{dt},\ U_C = \frac{q}{C}$$

Kirchhoff: $$L\frac{di}{dt} + Ri + \frac{q}{C} = 0$$

$$i = \frac{dq}{dt} \Rightarrow \boxed{L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 0}$$

Nghiem: $$q(t) = q_0 e^{-\frac{Rt}{2L}} \cos\left(\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}\, t\right),\ q_0 = V_0 C$$

Tìm R (L, C ña biet) ñe nang löông tieu hao cua mach co van toc cho tröôc: $q/q_0 = 0.01$ vôi t = 0.05s, L = 5H, C = 10^{-4} F

LÔI GIAI VÍ DU THÖC TE

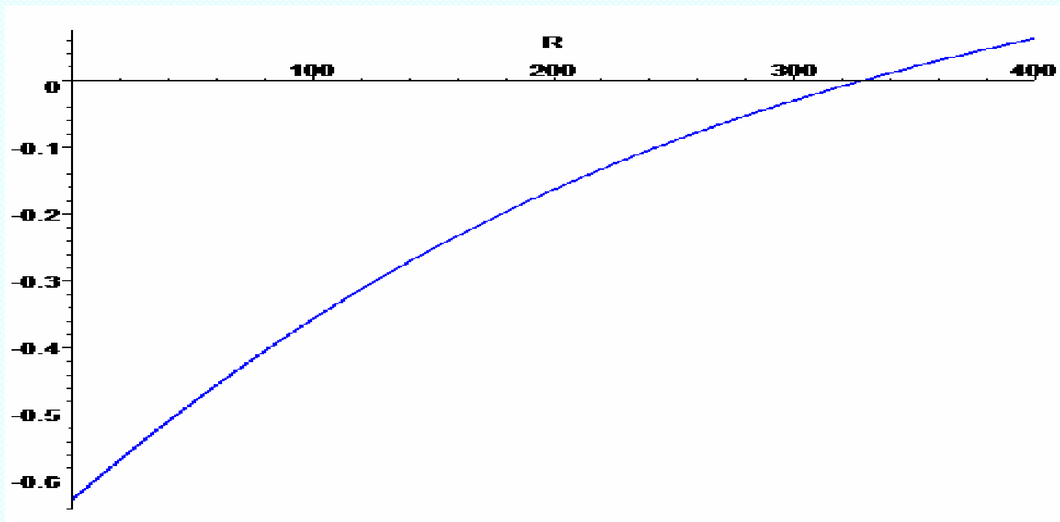
Bien ñoi phöông trình thu ñöôc (an R)

$$f(R) = e^{-\frac{Rt}{2L}} \cos\left(\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}\, t\right) - \frac{q}{q_0} = 0$$

$$\Leftrightarrow \boxed{f(R) = e^{-0.005R} \cos\left[\sqrt{2000 - 0.01R^2} \cdot (0.05)\right] - 0.01 = 0}$$

Khoang cach ly nghiem: R $\in$ [0, 400 Ω] ($2000 - 0.01R^2 \geq 0$)

Giai thöc te: Ñoi thò



❖P/p chia ñoi (n = 21)

❖P/p Newton

$\Rightarrow$ R = 328.1515 Ω